The First Challenge to Economic Rationality: Revisiting Allais Paradox

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Abstract

Allais Paradox is the first revealed violation of the underlying tenets of the theory of expected utility which has been the basic reference on economic behavior and economic decision making under risk since it was first suggested in 1944 by John von Neumann and Oskar Morgenstern. What makes Allais Paradox special is the paradox does not only violate the basic tenets of the theory of expected utility but also violates the independence axiom which is known as the heart of it. This violation provides evidence that adding a third alternative to the existing preferences matters. The aim of the study is to explain the first challenge to economic rationality; Allais Paradox and to show how the independence axiom which is known as the heart axiom of the theory of expected utility is violated. By revealing the paradox and the axiom; it is also aimed to call attention to the importance of this violation with regard to asset and insurance markets for future research.

Keywords: Rationality, Allais Paradox, Axioms, Independence Axiom, The Theory of Expected Utility, Prospect Theory.

INTRODUCTION

Still being a basic reference in economics research; the theory of expected utility examining economic behavior under risk and uncertainty was suggested by John von Neumann and Oskar Morgenstern in 1944. The theory of expected utility states that individuals act or make decisions or prefer an option considering the expected utilities of the alternatives. In fact, this statement is the finale ring of a long chain. Mathematicians Blaise Pascal and Pierre de Fermat were involved in computing the "expected value" of all kind of chance games in the 17th century. In 18th century Bernoullis were included in the research. After Nicolaus Bernoulli discovered that individuals are interested in more than just expected value but unfortunately couldn't clarify what it was; his cousin Daniel Bernoulli suggested that it is the "utility" considered. Daniel Bernoulli revealed this by St. Petersburg Paradox.

The scenario of the paradox is based upon tossing a fair coin where 1 dollar is doubled until the first tail appears. The basic idea behind the paradox is the unwillingness of individuals to pay higher amounts to take such a gamble which offers an infinite gain. After it is provided evidence that rather than calculating expected values individuals consider utility; John von Neumann and Oskar Morgenstern revealed that it is the "expected utility" taken into consideration. The theory of expected utility has had great currency since it was first suggested in 1944 and has still been the basic reference in the field.

The expected utility theory has a consistent structure in itself which is the main reason for it to be respected. The theory explains economic behavior or economic decision making within the framework of a range of assumptions so called axioms which are transitivity, substitution,

monotonicity, Archimedean, unequal-probability, independence, invariance, continuity and completeness (complete-ordering). Among all; the independence axiom is known as the key or the heart of the expected utility theory. What makes these axioms special for the theory is the connection with the theoretical definition of economic rationality. An economic decision or a preference or behavior is stated as rational as long as it satisfies expected utility theory's axioms.

In 1953 Nobel winner French economist Maurice Allais showed the violation of the independence axiom using a set of problems. Since the independence axiom is stated as the heart of the theory; Allais finding which is known as Allais Paradox in the literature was a strike at the heart of the theory.

This study reveals a critique of economic rationality in terms of the violation of the independence axiom of the expected utility theory within the concept of Allais Paradox. The rest of the paper is organized as follows. After a detailed definition in introduction part; the independence axiom of the expected utility theory is explained in section 1. Section 2 presents Allais Paradox and how the independence axiom of the expected utility theory is violated. In section 3 the relevance with the prospect theory explained. Final section concludes.

1.THE INDEPENDENCE AXIOM OF THE THEORY OF EXPECTED UTILITY

The independence axiom is regarded as the key axiom of the expected utility theory. According to the expected utility theory's independence axiom when they are making a choice between the alternatives decision makers ignore the common parts suggested.

When a rational individual is asked to make a choice between a lottery that suggests outcomes P and R with the probability values α and $(1 - \alpha)$ respectively and the lottery that suggest the outcomes Q and R with the probability values α and $(1 - \alpha)$ respectively

$$\alpha P + (1 - \alpha)R \text{ and } \alpha Q + (1 - \alpha)R;$$

the independence axiom states that preference should not depend on the "common consequence" R (Abdellaoui, 2002):

$$(P \square Q, \alpha \in [0, 1]) \rightarrow (\alpha P + (1 - \alpha)R \square \alpha Q + (1 - \alpha)R.$$

For example if x, y, z are different outcomes and the preference between x and y is x is preferred to y; x $\square$ y and then when the individual is asked to make a choice between x and y including a common outcome z; the preference order is preserved: x+z $\square$ y+z. It is possible to show the same preference order using the corresponding probability values. So if $\alpha x' + (1 - \alpha)z'$ and $\alpha y'+(1 - \alpha)z'$ the alternatives asked to be choice by an individual, the common part $(1 - \alpha)z'$ is ignored and the individual considers αx and αy.

If a lottery L_1 suggests outcomes A and C with the probability values of P and (1-P) respectively, and lottery L_2 suggests outcomes B and C with the probability values of P and (1-P) respectively, and if an individual prefers A to B; then it is concluded that L_1 is preferred to L_2 according to the expected utility theory's independence axiom revealing that the common outcome is ignored (Henderson and Quandt, 1980).

Independence is the axiom that leads the preference function to be linear in the probabilities(Machina, 1982). According to the independence axiom indifference curves are parallel straight lines; if

$$p \sim p', \text{ then } p \sim \alpha p + (1 - \alpha)p' \sim p',$$

 if not then

$$p' \sim p \text{ and but } 1/2 \ p' + 1/2 \ p'' \square 1/2 \ p+ 1/2 \ p'' \text{ (Levin, 2006).}$$

2.ALLAIS PARADOX

Allais presented that the preferences between the suggested hypothetical situations violate the independence axiom of the theory of expected lottery leading to common consequence and common ratio effects.

A: 100 m for sure

B: 10% 500 m

89%100 m

1% 0

A': 11% 100 m 89% 0

B': 10% 0 500 m 90% 0

C: 100 m for sure

D: 98% 500 m 2% 0

C': 1% 0 100 m 99% 0

D': 0.98% 500 m 99.02% 0

For A, B, A', B' and C, D, C', D'; A and B have winning 100 million with probability value of 0.89 in common as a common consequence which is removed in A' and B' where A' should be preferred to B' if A is preferred to B and C and D have probability value of 0.98 in common where C' should be preferred to D'; if C is preferred to D according to the expected utility theory whereas A over B, B' over A' , C over D and D' over C' are chosen by the majority of subjects violating the expected utility theory's independence axiom contradicting with the theoretical rationality(Allais, 1953; Andreoni and Sprenger, 2010; Kahneman and Tversky, 1979).

Different forms and conditions of Allais paradox and the independence axiom's systematic violations are shown by several research (Huck and Müller, 2012; Blavatskyy, 2013, 2010; Day and Loomes, 2010; Da Silva et. al. ,2013; Cherry et. al. , 2007; Humphrey and Verschoor, 2004; Birnbaum, 2004; Fan, 2002; Groes et. al. , 1999; Cubitt et al. , 1998; Beattie and Loomes, 1997; Finkelshtain and Feinerman, 1997; Wu and Gonzalez, 1996; Burke et. al. , 1996; Harrison, 1994; Starmer and Sugden, 1991, 1993; Carlin, 1990, 1992; Harless, 1992a, 1992b; Starmer, 1992; Loomes, 1991; Battalio et. al. , 1990; MacDonald and Wall, 1989; Conlisk, 1989; Camerer, 1989; Machina, 1987; Holt, 1986; Hagen, 1979; Kahneman and Tversky, 1979; Moskowitz, 1974; Slovic and Tversky, 1974; MacCrimmon and Larsson, 1979; Raiffa, 1968; Morrison, 1967; MacCrimmon, 1967.

Violation of the independence axiom is known as the common consequence effect, common consequence problem, common ratio effect.

3. ALLAIS PARADOX AND THE PROSPECT THEORY

In 1979 Daniel Kahneman and Amos Tversky presented "prospect theory" in order to explain decision making under risk considering the systematic contradictions with the basic tenets of the expected utility theory. Unlike the expected utility theory; prospect theory examines gains and losses rather than final asset position and suggests the value function which is concave for gains and convex and steeper for losses referring to loss aversion means sensitivity towards losses. Kahneman and Tversky also suggested the weighting function as well where it is shown that the decision weights are generally lower than the corresponding probabilities except in the range of low probabilities.
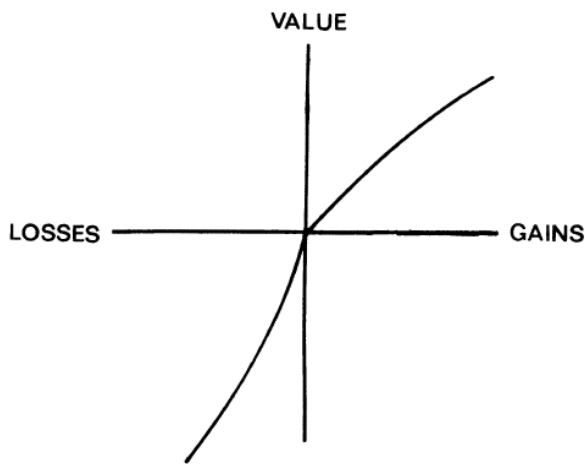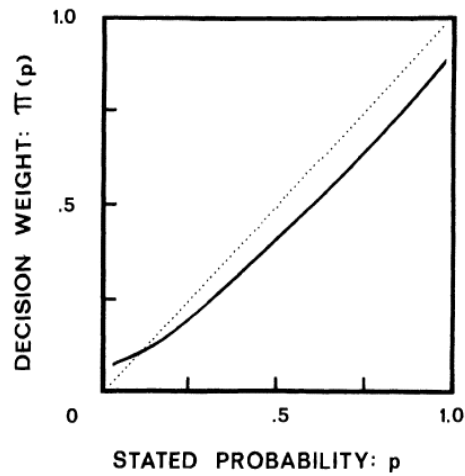
Figure 1: Hypothetical Value Function

Figure 2: Hypothetical Weighting Function

Source: Kahneman and Tversky, 1979:279.

Source: Kahneman and Tversky, 1979:283.

Kahneman and Tversky presented a pair of choice problems in 1979 which is a variation of Allais' example and they once more provided evidence that individual behavior violate the independence axiom of the expected utility theory in a manner which was shown by Allais in 1953. N denotes the total number of the respondents, and the percentage of the chosen option is given in brackets. The authors used Israeli currency.

Problem 1:

Choose between

A: 33% 2500

B: 2400 for sure

66% 2400

 1% 0

 N=72 [18]

[82]

According to the given information; 72 respondents are asked to make a choice between the options A and B. Option A suggests outcome 2500 with the probability value of 33%, the outcome 2400 with the probability value of 66% and 0 with the probability value of 1%. On the other hand option B suggests 2400 for sure. It is seen that 82% of the respondents choose option B where the offered amount is less but certain.

Problem 2:

Choose between

C: 33% 2500

D:34% 2400

67% 0

66% 0

N =72 [83]*

[17]

According to the given information; this time 72 respondents are asked to make a choice between the options C and D. Option C suggests the outcome 2500 with the probability value of 33% and nothing with the probability value of 67%. On the other hand, outcome 2400 is suggested with the probability value of 34% and nothing with the probability value of 66% by option D. It is seen that option C is mostly chosen where the higher outcome is offered with a lower probability value.

The preference relations according to the options A and B and options C and D demonstrate how the independence axiom of the expected utility theory is violated as it was done in Allais Paradox:

the preference relation between A and B

u(2400)>0.33u(2500)+0.66u(2400)+0.01u(0)

0.34u(2400) >0.33u(2500)

the preference relation between C and D

0.33u(2500)+0.67u(0)>0.34u(2400)+0.66u(0)

0.33u(2500)>0.34u(2400)

Kahneman and Tversky also provided evidence that the individual behavior violates the expected utility theory's independence axiom using gambles involve only two outcomes.

CONCLUSION

As Allais Paradox sheds light on the understanding of economic decision making and the related phenomenons such as the certainty effect, risk aversion, probability weighting, framing effect, risk seeking, consideration of gains and losses, quasi-hyperbolic discounting and sensitivity towards losses; it has also implications regarding to the asset markets.

Ignoring the common consequence or common alternative or common outcome between the alternatives is an issue to be called into question in order to yield accurate policies. This behavioral tendency also effects stock markets referring to portfolio diversification. Related to the attitudes towards risk insurance marketing should pay regard to Allais Paradox and the violation of the independence axiom of the expected utility theory and its implications as well.

The demonstration of the violation of the independence axiom within the concept of Allais Paradox has considerable indications regarding to gambling and addiction to gambling with its link to risk aversion and risk seeking behavior.

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